



Student name: _____ Student number: _____

1. (4 points) Determine whether each of the following statements is true or false

- (i) To show that p implies q vacuously, one uses the fact the q is true and the proof follows.
- (ii) $\{10, 11, 12, 13, \dots\} \subset \{x : x \text{ is an integer greater than } 9\}$.
- (iii) Suppose that A and B are two finite sets. Then $|A - B| = |A| - |B|$.
- (iv) Suppose that $f : A \rightarrow B$ is a one to one function and that $S \subseteq A$. Then $|S| = |f(S)|$.

2. (2+1+2 points) Answer the following questions:

- (i) Given that $\lceil 2x - 3 \rceil = 4$. The solution set for x is
- (ii) Let $f(x) = \lceil x^2 \rceil$. Then $f(\mathbb{R})$ is
- (iii) Suppose that $f : A \rightarrow B$ is a surjective function and that $|A| = 7$. Then $|B|$ is

3. (2 point) Suppose that the universal set $U = \{a_1, a_2, a_3, \dots, a_n\}$. Find

- (i) The set with bit string that has exactly one 0.
- (ii) The set that is represented by the bit string not containing the bit 1.

4. (3 points) Let A and B be two sets. Show that if $A \subseteq B$, then $\overline{B} \subseteq \overline{A}$.

5. (2+1 points) Let $A_i = \{\frac{1}{i}, i-1, 1-i\}$. Find $\bigcup_{i=1}^{\infty} A_i$ and $\bigcap_{i=1}^{\infty} A_i$.

6. (4 points) Show that for an integer n , if n^2 is even then $n + 1$ is not a multiple of 4.

7. (4 points) Show that the function $f : (1, \infty) \rightarrow (-\infty, 0)$ given by $f(x) = \frac{1}{(1-x)^3}$ is a bijective.