A

solutions

PSUT

Department of Basic Sciences

Calculus II - Second Exam- Semester (2) - 2016/2017

Name: Student's Number:

Section Time: Teacher's Name:

Question No.	1	2	3	4	5
Answer	A	C	D	B	D

- Write the correct answer in the box above. [2 pts each]

1. The point $(-\sqrt{2}, \sqrt{2})$ in $xy-$coordinates is equal in polar coordinates to
(A) $[-2, -\frac{\pi}{4}]$ (B) $[2, \frac{5\pi}{4}]$ (C) $[-2, \frac{\pi}{4}]$ (D) $[-2, \frac{3\pi}{4}]$

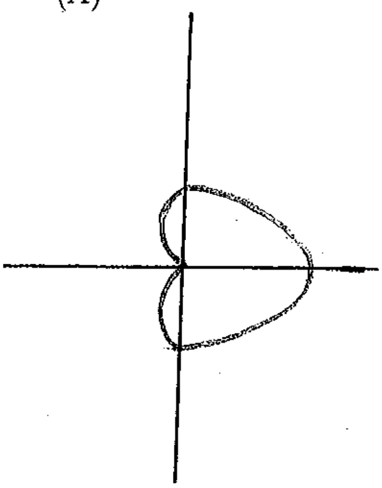
2. The parametric equations $x = \cos t$ and $y = \cos(2t)$, where $0 \leq t \leq \pi$, represent
(A) a line with start point $(1,1)$. (B) a circle with start point $(1,1)$.
(C) a parabola with start point $(1,1)$. (D) a semicircle with start point $(0,0)$.

3. The curve given by $x = \frac{1}{3}t^3 - \frac{\pi}{8}t^2$ and $y = \cos t$, where $t \in [0, 2\pi]$, has horizontal tangents at
(A) $t = 0$ and $t = \frac{\pi}{4}$ (B) $t = \frac{\pi}{4}$ only (C) $t = 0$ only (D) None

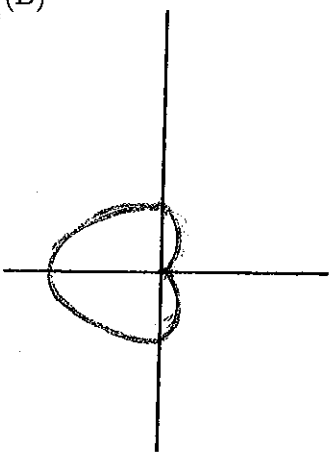
4. The curve $r^4 = |\theta|$ is symmetric about
(A) the $x-$axis and the $y-$axis only. (B) the $x-$axis and the origin only.
(C) the origin only. (D) the $y-$axis, and the origin only.

5. The polar curve given by $r = 6 - 6\sin\theta$ with $0 \leq \theta \leq 2\pi$ is

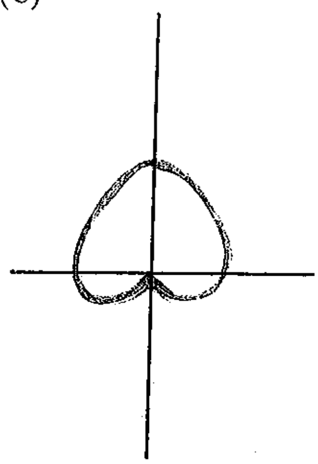
(A)



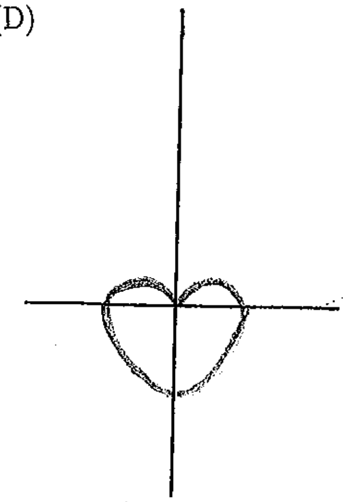
(B)



(C)



(D)



B

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Department of Basic Sciences

Calculus II - Second Exam- Semester (2) - 2016/2017

Name: Student's Number:

Section Time: Teacher's Name:

Question No.	1	2	3	4	5
Answer	D	B	B	C	B

- Write the correct answer in the box above. [2 pts each]

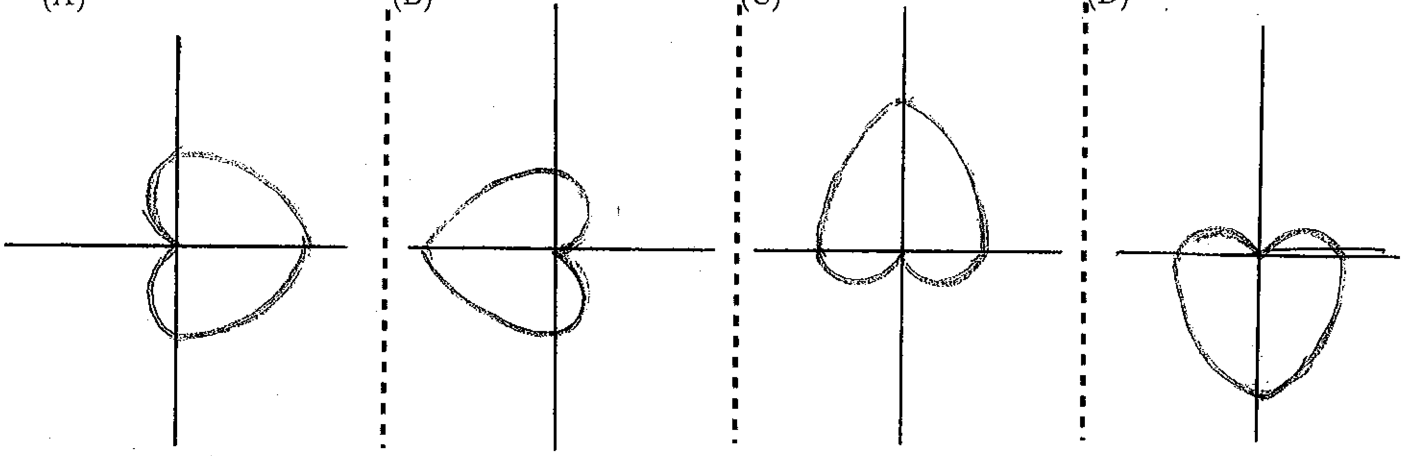
1. The point $(\sqrt{2}, -\sqrt{2})$ in $xy-$coordinates is equal in polar coordinates to
(A) $[-2, -\frac{\pi}{4}]$ (B) $[2, \frac{5\pi}{4}]$ (C) $[-2, \frac{\pi}{4}]$ (D) $[-2, \frac{3\pi}{4}]$

2. The parametric equations $x = \sin t$ and $y = \cos(2t)$, where $0 \le t \le \pi$, represent
(A) a line with start point $(0,1)$. (B) a parabola with start point $(0,1)$.
(C) a circle with start point $(0,1)$. (D) a semicircle with start point $(0,1)$.

3. The curve given by $x = \frac{1}{3}t^3 - \frac{\pi}{8}t^2$ and $y = \cos t$, where $t \in [0, 2\pi]$, has vertical tangents at
(A) $t = 0$ and $t = \frac{\pi}{4}$ (B) $t = \frac{\pi}{4}$ only (C) $t = 0$ only (D) None

4. The curve $|r| = \theta^3$ is symmetric about
(A) the $x-$axis and the $y-$axis only. (B) the $x-$axis and the origin only.
(C) the origin only. (D) the $y-$axis, and the origin only.

5. The polar curve given by $r = 6 - 6\cos\theta$ with $0 \le \theta \le 2\pi$ is
(A) (B) (C) (D)



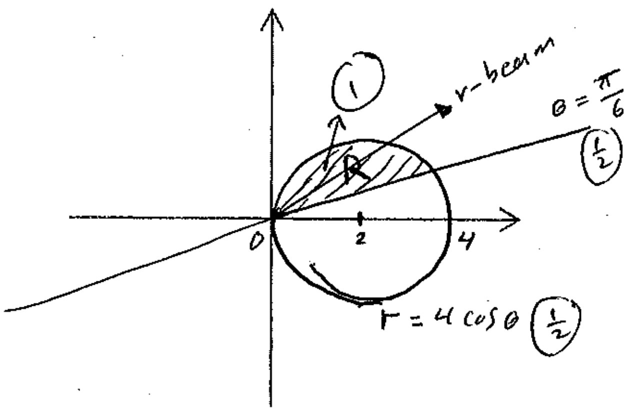
6. Find the intersection points between the curves $r = 2$ and $r = 4\sin\theta$, where $\theta \in [0, 2\pi]$. [4 pts]

- Since $r = 2 \neq 0$, the origin is not an intersection point (1)
- $4\sin\theta = 2 \Rightarrow \sin\theta = \frac{1}{2} \Rightarrow \theta = \frac{\pi}{6}, \frac{5\pi}{6}$ (2)

The points of intersection are $[2, \frac{\pi}{6}]$ and $[2, \frac{5\pi}{6}]$ (1)

7. Consider the curves $\theta = \dfrac{\pi}{6}$ and $r = 4\cos\theta$. [2 pts each]

a. Sketch the curves in the same xy-plane, and shade the region R in part b.



b. Compute the area of the region R located in the first quadrant and lies between the curves .
Set the integral only. Do not compute it.

$$\text{Area} = \frac{1}{2}\underbrace{\int_{\theta=\frac{\pi}{6}}^{\theta=\frac{\pi}{2}}}_{(1)} \underbrace{[(4\cos\theta)^2 - 0^2]}_{(1)}\, d\theta$$

8. Find $\lim_{n\to\infty} \sqrt[n^2]{n^2 \cdot 5^{n^2-n}}$. [4 pts]

$= \lim_{n\to\infty} \sqrt[n^2]{n^2} \cdot \lim_{n\to\infty} \sqrt[n^2]{5^{n^2-n}} = \lim \sqrt[n^2]{n^2} \cdot \lim 5^{\frac{n^2-n}{n^2}}$

$= \lim_{k\to\infty} \sqrt[k]{k} \cdot \lim_{n\to\infty} 5^{1-\frac{1}{n}} = 1 \cdot 5$ ~~Ain~~

$= 5$.

$\boxed{\text{each limit 2 marks}}$

where $k = n^2$ and as $n \to \infty$, $k \to \infty$.

9. Determine whether the improper integral $\int_{\frac{\pi}{3}}^{\infty} x^{99} \cdot (2+\cos x)^5 \, dx$ is convergent or divergent. Show all your work. [3 pts]

By Comparison Test: $\forall x \geq \frac{\pi}{3}$

$\cos x > -1$

$2 + \cos x > 1$

$(2+\cos x)^5 > 1^5 = 1$

$x^{99} \cdot (2+\cos x) > x^{99}$ (1)

Since $\int_{\pi/3}^{\infty} x^{99} \, dx \overset{\text{or}}{=} \int_{\pi/3}^{\infty} \frac{1}{x^{-99}} dx$ is divergent by P-Test (1)

by Comparison test $\int_{\pi/3}^{\infty} x^{99} \cdot (2+\cos x)^5 dx$ is also divergent (1)