Solution

Department of Basic Sciences

Discrete Math II - Second Exam - Semester (2) - 2017/2018

Name: Student's Number:

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1. Consider the Boolean function $F(x,y,z) = \bar{x}(\bar{x} + yz) + \overline{xy}$.

a. Find the complement function $\bar{F}(x,y,z)$ of F. [1]

$$\bar{F} = x + [x \cdot (\bar{y} + \bar{z})] \cdot (x \cdot y) = x \cdot (x y) = x \cdot y$$

b. Find the duality function $F^d(x,y,z)$ of F. [1]

$$F^d = [\bar{x} + (\bar{x} \cdot (y+z))] \cdot (\overline{x+y}) = \bar{x} \cdot (\bar{x} \cdot \bar{y}) = \bar{x} \cdot \bar{y}$$

c. Use identity approach with detailed steps to show that $F(x,y,z) = \bar{x} + \bar{y}$. [2]

$$
\begin{aligned}
F(x,y,z) &= \bar{x} \cdot (\bar{x} + yz) + \overline{xy} \\
&= \bar{x} + \overline{xy} && \text{absorbtion} \\
&= \bar{x} + (\bar{x} + \bar{y}) && \text{De Morgan} \\
&= (\bar{x} + \bar{x}) + \bar{y} && \text{Associative} \\
&= \bar{x} + \bar{y} && \text{Idempotent}
\end{aligned}
$$

d. Represent $F(x,y,z)$ as the sum of products. [2]

By c) $F(x,y,z) = \bar{x} + \bar{y} = \bar{x} \cdot \overbrace{(y + \bar{y})}^{1} + \bar{y} \cdot \overbrace{(z + \bar{z})}^{1}$

$$
\begin{aligned}
&= \bar{x} y + \bar{x}\bar{y} + \bar{y} z + \bar{y}\bar{z} \\
&= (\bar{x} y + \bar{x}\bar{y})(z + \bar{z}) + (\bar{y} z + \bar{y}\bar{z}) \cdot (x + \bar{x}) \\
&= \bar{x} y z + \bar{x} y \bar{z} + \underline{\bar{x}\bar{y} z} + \underline{\underline{\bar{x}\bar{y}\bar{z}}} + \bar{y} z x + \underline{\bar{y} z \bar{x}} + \bar{y}\bar{z} x + \underline{\underline{\bar{y}\bar{z}\bar{x}}} \\
&= \bar{x} y z + \bar{x} y \bar{z} + \bar{x}\bar{y} z + \bar{x}\bar{y}\bar{z} + x \bar{y} z + x \bar{y} \bar{z}
\end{aligned}
$$

2. Find $F(x,y)$ as a Boolean function of x and y, given that [2]

$$F(1,1) = 0,\ F(1,0) = 1,\ F(0,1) = 1,\ F(0,0) = 0.$$

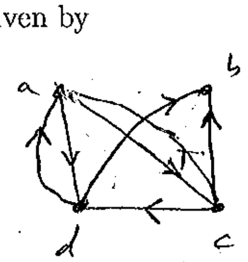
$F(1,0) = 1 \longrightarrow x\bar{y}$

$F(0,1) = 1 \longrightarrow \bar{x} y$

$\therefore F(x,y) = x\bar{y} + \bar{x} y$

3. A digraph G with ordered set of vertices $\{a, b, c, d\}$ has adjacency matrix given by

$$A_G = \begin{bmatrix} 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 0 \\ 1 & 1 & 0 & 1 \\ 1 & 1 & 0 & 0 \end{bmatrix}.$$



Answer the following questions showing your explanation.

a. Is the graph simple or multiple? [1]

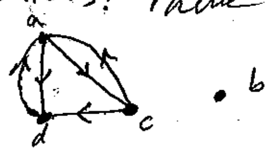
It is simple because it 0-1 matrix with all zeros on main diagonal (no loops).

b. Is the graph weakly connected? [1]

Yes because, the undirect graph G is Connected.

c. Is the graph strongly connected? Find the number of strongly connect components. [3]

No, because there is no path from b to c for example.
For strongly Components: There are 2 strongly Components; namely



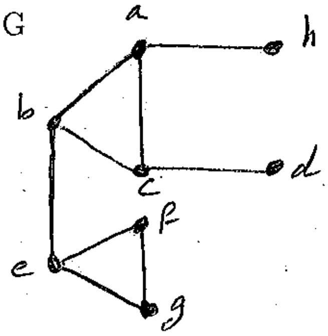
d. What is the length of shortest path from d to c, and how many such paths? Give your answer using A_G. [2]

$$A_G^2 = \begin{bmatrix} 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 0 \\ 1 & 1 & 0 & 1 \\ 1 & 1 & 0 & 0 \end{bmatrix} \cdot \begin{bmatrix} 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 0 \\ 1 & 1 & 0 & 1 \\ 1 & 1 & 0 & 0 \end{bmatrix}$$

$$= \begin{array}{cc} & c \\ d & \begin{bmatrix} & & 1 & \end{bmatrix} \end{array}$$

So, the shortest path is of length 2, and there is only <u>one</u> such path from d to c.

4. Consider the graph G



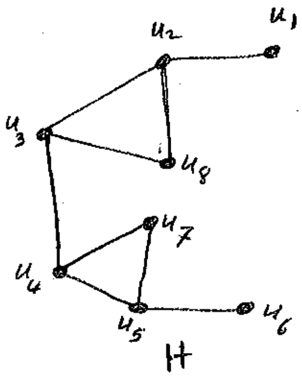
a. Find the cut vertices of this graph. [1]

a, c, b, e

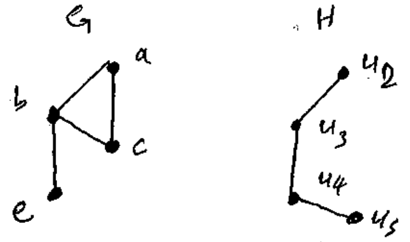
b. Find the cut edges of this graph. [1]

ah, cd, be,

c. Give a reason why G is not isomorphic to the graph H given by [3]



Consider the subgraphs of degree 3 in G & H

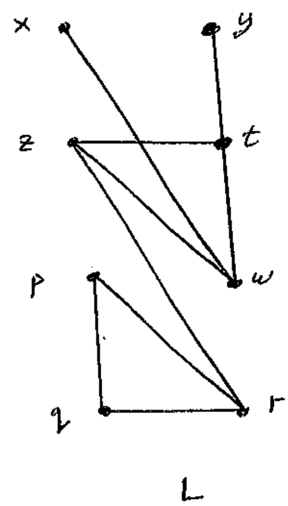


They are not isomorphic since
* That of G has a vertex of degree 3 (b) but that of H has no vertex of degree 3.

or * That of G has a simple circuit while that of H has no simple circuits.

* or They have different number of edges.

d. Show that G is isomorphic to the graph L given by [3]



The bijection f given by

G	a	b	c	d	e	f	g	h
L	t	z	w	x	r	p	q	y

edgewise: f maps each edge of G onto one edge of L.

So, f is an isomorphism.

or

G	a	b	c	d	e	f	g	h
	w	z	t	y	r	p	q	x

5. Let G be a simple undiredted graph whose adjecency matrix contains a row with all entries are zeros. Show that the complement graph $\bar{G}$ is connected. [2]

$A_{\bar{G}}$ is obtained from A_G by replacing 0's with 1's and 1's with 0's in A_G. Thus, the row with 0's entries in A_G becomes a row with 1's in $\bar{A}_G$. Thus, there is a vertex λ that is adjecent to all vertices of $\bar{G}$. Thus between any two vertices of $\bar{G}$ there is a path of length 1 or a path with length 2 (passing through the vertex λ). Therefore, $\bar{G}$ is connected.

Extra Space