Discrete Mathematics

Final Exam.
Feb.2014

اسم المدرس: رجاء

اسم الطالب:

وقت المحاضرة:

رقم الطالب:

Question	1	2	3	4	5	6	7	8	Total
Your Score									
Maximum Score	6	4	4	2	6	6	7	5	40

Q.1: Use **Induction** to prove the statement:
$P(n) : 7^n - 1$ is divisible by 6 , for all $n \geq 1$.

* Basic Step $P(1): 7^1 - 1 = 6$, which is divisible by 6 (1)

* Induction Step : Assume $P(k)$ is true. (1)

$P(k): 7^k - 1$ is divisible by 6,
$7^k - 1 = 6m,\ m \in \mathbb{Z}$. (1)

Want to show $P(k+1)$ is true.

$P(k+1) \Rightarrow 7^{k+1} - 1$ is divisible by 6
i.e $7^{k+1} - 1 = 6h,\ h \in \mathbb{Z}$. (1)

Proof:
$$7^{k+1} - 1 = 7 \cdot 7^k - 1$$
$$= \underbrace{7^k - 1} + 6(7)^k$$
$$= 6m + 6(7)^k$$ (2)
$$= 6(\underbrace{m + 7^k}_{\text{integer}}) \quad \therefore \text{divisible by } 6$$

Q.2: i- **Disprove:** If $a \equiv b (mod\ m)$ and $c \equiv d (mod\ m)$ then $a^c \equiv b^d (mod\ m)$
Where a,b,c,d,m are integers c,d positive , m>1

~~Student's answer~~

(2)

ii- Show that the following statement is NOT tautology: $(p \wedge \neg q) \rightarrow r$

$(p \wedge \neg q) \longrightarrow r$

$-p \vee q \vee r$

p : F
q : F
r : F

(2)

Q.3: Consider the following relation on real numbers: x R y iff x < y + 2.
Determine whether it satisfies the following properties(justify your answer)

Reflexive (✓)...... $x < x + 2, \forall x \in \mathbb{R}$ ...... (1)

Symmetric (x)...... $(1, 3) \in R$ ... $1 < 5$, ... $(3, 1) \notin R$ ... $3 \nless 3$ ... (1)

Transitive (x)...... $(5, 4) \in R$ & $(4, 2.1) \in R$ but $(5, 2.1) \notin R$ (1)

Anti-symmetric (x)...... $(1.5, 0)$, $(0, 1.5)$ ...... (1)

Q.4: Complete the following where x is a real number
i)If x is not an integer then $\lfloor x \rfloor - \lceil x \rceil = -1$ (1)

ii)If x is an integer then $\lfloor x \rfloor + \lceil x \rceil = 2x$ (1)

Q.5 (a)Evaluate the expression which is given in **postfix notation:**

9 7 − 3 + 2 8 4 + − 1 5 ^ × ÷

2 3 ÷ 2 12 − 1 × ÷

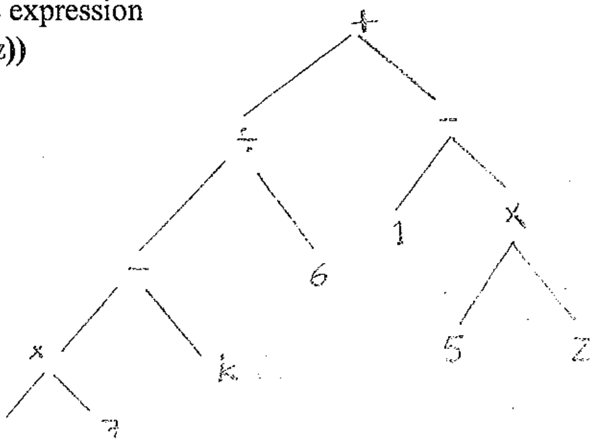
5 (−10) 1 × ÷

5 (−10) ÷

$= -\frac{1}{2}$

2

(b) Construct the tree of the algebraic expression
$(((h \times 7) - k) \div 6) + (1 - (5 \times z))$



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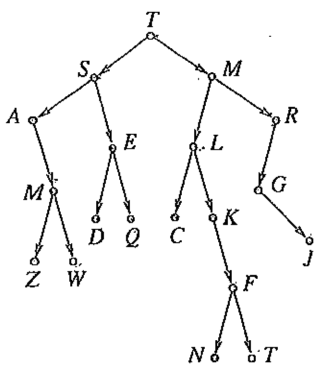
(c) Consider the tree whose digraph is shown and the accompanying list of words:
Give the sentence that results from doing an **inorder** search of the tree



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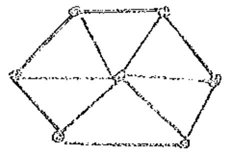
A Z M W S D E Q T C L K N F T M G J R

Q.6: i-How many vertices does the bipartite graph $K_{8,7}$ have ? 15 (1)

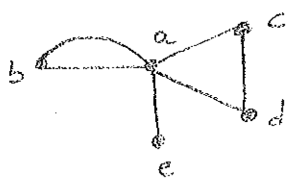
ii-Give a sketch of W_6?



(1)

iii-How many edges does the complete graph K_n $\frac{n(n-1)}{2}$ (2)

iv- An undirected graph with vertices **a,b,c,d,e**
Given that the degree of the vertices respectively is **5,2,2,2,1**
I-How many edges does the graph have? 6 edges (1)
II-Draw such a graph. (1)



Q.7: (a)Solve the recurrence relation $U_{n+1} = U_n - 4$, $U_1=2$ "simplify your answer"

$U_n = 2 + -4(n-1) \longrightarrow U_n = 6 - 4n$

(1) (1)

(b) Solve the recurrence relation $U_n=5U_{n-1}+14U_{n-2}$,where $U_1=1$, $U_2=61$

(1) $x^2 = 5x + 14$

$x^2 - 5x - 14 = 0$

$(x-7)(x+2) = 0$

(1) $S_1 = 7$, $S_2 = -2$

$U_n = u(S_1)^n + V(S_2)^n$

$\boxed{n=1}$ $[1 = u(7) + V(-2)] \times 2$ (0.5)

$\boxed{n=2}$ $61 = u(49) + V(4)$ (0.5)

$+ \quad 2 = u(14) + V(-4)$

$63 = 63u$ $\quad u = 1$ (0.5)

$V = 3$ (0.5)

$U_n = (7)^n + 3(-2)^n$ (1)

Discrete Mathematics

Final Exam.

August, 2011

اسم الطالب: اسم المدرس:

رقم الطالب: وقت المحاضرة:

Question	1	2	3	4	5	6	7	Total
Your Score								
Maximum Score	/8	/7	/2	/5	/8	/9	/6	45

NOTE: "SHOW CLEARLY YOUR WORK"

Q.1:

a) Find the negation of " $\exists x \forall y : (p(x,y) \rightarrow q(x,y))$ "

b) Find the contrapositive of " $\neg p \rightarrow (r \rightarrow (q \vee s))$ "

c) Disprove If ac≡bc(mod m) where a,b,c ,m ∈ Z,m≥2 then a≡b(mod m)

d) determine the truth of

i- $\exists x \forall y : x < y^2 + 1 \quad x, y \in Z$

ii- $\forall x \exists y : xy^2 = 1 \quad x, y \in R^+$

Q.2: a)Find the matrix X_{2x2} in each case:

(i) $$\begin{pmatrix} 2 & 1 & -3 \\ 0 & 5 & 1 \end{pmatrix}\begin{pmatrix} 4 & 1 \\ 0 & 3 \\ 1 & 2 \end{pmatrix} = \begin{pmatrix} 2 & 6 \\ -2 & -1 \end{pmatrix}^T - \frac{1}{2}X$$

(ii) $$\begin{pmatrix} 4 & -3 \\ 9 & -7 \end{pmatrix}X = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$$

(b)Consider the Boolean matrices,find:

$$\begin{pmatrix} 0 & 1 & 1 \\ 1 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix}^{[2]} \wedge \begin{pmatrix} 0 & 0 & 0 \\ 1 & 1 & 0 \\ 1 & 0 & 0 \end{pmatrix}$$

Q.3:Find all solutions for: $\lceil x + 4 \rceil = -1$ where x is a real number

Q.4: Prove by induction $P(n): \frac{1}{1(3)} + \frac{1}{3(5)} + \frac{1}{5(7)} \ldots\ldots\ldots\ldots\ldots + \frac{1}{(2n-1)(2n+1)} \ldots = \frac{n}{2n+1}, \forall n \geq 1$

Q.5:(a)Evaluate the expression which is given in **postfix notation:**

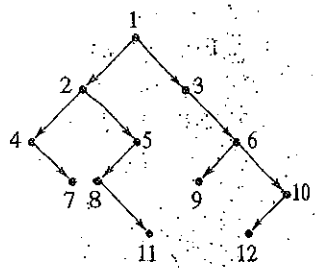
$$7 \ 2 \ - \ 3 \ + \ 2 \ 3 \ 4 \ + \ - \ 1 \ 5 \ \wedge \ \times \ \div$$

(b) Construct the tree of the algebraic expression

$$(((y \times 4) - z) \div 7) + (8 - (5 \times w))$$

(c) Consider the tree whose digraph is shown and the accompanying list of words:

Give the sentence that results from doing an **inorder** search of the tree

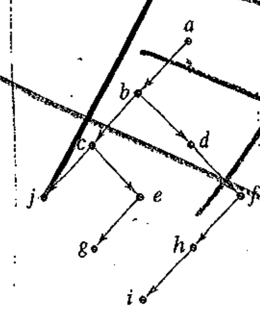


1. ONE	7. I
2. COW	8. A
3. SEE	9. I
4. NEVER	10. I
5. PURPLE	11. SAW
6. NEVER	12. HOPE

(d) Consider the digraph of the labeled binary positional tree shown.

If this tree is the binary form B(T) of some tree T,draw the digraph of tree T

B(T) **T**



Faculty of Engineering
Department of Basic Sciences

Discrete Math I
June 9, 2015
Final Exam

Student name: ____________________ Student number: ____________________

1. (3 marks) Show that if $x + y \geq 2$, where x and y are two real numbers, then $x \geq 1$ or $y \geq 1$.

2. (3 marks) Use predicates, quantifiers and logical connectives to express the statement "There is somebody who has no friends besides himself" as a logical expression.

3. (3 marks) Show that the hypotheses

- $(\neg r \vee \neg f) \rightarrow (s \wedge l)$
- $s \rightarrow t$
- $\neg t$

lead to the conclusion r.

4. (3 marks) Use mathematical induction to show that

$$n! < n^n, \quad \text{for all integers } n \geq 2$$

$2!$ $\quad 2^2$

2 $\quad 4$

$(k+1)! <$

5. (4 marks) Solve the following recurrence relation together with the initial conditions given.

$$a_n = 8\,a_{n-1} - 16\,a_{n-2} \text{ for } n \geq 2,\ a_0 = 4,\ a_1 = 20$$

6. (2 marks) Show that the function $f : [2, \infty) \to \mathbb{R}$ given by $f(x) = \frac{x}{x-1}$ is one-to-one.

9. (3 marks) Find the *in-degree* and *out-degree* of each vertex for the given directed multigraph G.

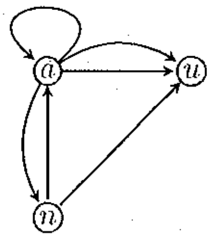


FIGURE 1. Graph G

10. (3 marks) Determine whether the graph H below is a *bipartite*? If so, complete H into a complete *bipartite*.

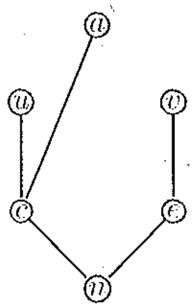


FIGURE 2. Graph H

11. (1 mark) Can a simple graph with seven vertices each of degree three exist? Justify your answer.

12. (3 marks) How many vertices and how many edges do each of the following graphs have?

(i) $K_{4,9}$

(ii) C_{200}

(iii) W_{10}

13. (2 marks) Given that the inverse of $A = \begin{bmatrix} 1 & 2 & 3 \\ 2 & 5 & 3 \\ 1 & 0 & 8 \end{bmatrix}$ is $A^{-1} = \begin{bmatrix} -40 & 16 & 9 \\ 13 & 1-t^2 & -3 \\ 5 & -2 & -1 \end{bmatrix}$. Find the value(s) of t.

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14. (2 marks) Let $A = \begin{bmatrix} 3 & -4 \\ 5 & -6 \end{bmatrix}$. Find A^{-1}.

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15. (3 marks) Let $A = \begin{bmatrix} 1 & 0 \\ 0 & 1 \\ 1 & 1 \end{bmatrix}$ and $B = \begin{bmatrix} 1 & 0 & 0 \\ 1 & 1 & 1 \end{bmatrix}$. Find

(i) $A \odot B$

(ii) $A \wedge B^t$ **Hint.** B^t is the transpose of B

(iii) $A \vee B^t$

Good luck :)

Discrete Mathematics

Summer Semester 2014/2015

Final Exam(Make-up) 17/8/2015

Student's Name:

ID:

Question	1	2	3	4	5	6	Total
Your Score							
Maximum Score	4	6	10	8	6	6	40

Q#1:(I) Find the negation of

$$\forall x \exists y (P(x,y) \rightarrow \neg Q(x,y))$$

(II) Determine whether the argument is valid or not:(show clearly your work)

$$((p \rightarrow \neg r) \wedge (\neg q \rightarrow p)) \rightarrow (\neg r \wedge q)$$

Q#2:(I) Solve the following recurrence relation:

$$a_n = a_{n-1} - 4\,, a_1 = 2$$

(II) Solve the recurrence relation

$$a_n = 5a_{n-1} + 14a_{n-2}, \quad a_1 = 1, a_2 = 61$$

Q#3:(I) If $A = \begin{pmatrix} -3 & -7 \\ 2 & 8 \end{pmatrix}$ $find\ 10A^{-1}$.

(II) Given that $\begin{pmatrix} 2 & x & -1 \\ 1 & 2 & 1 \\ -1 & -1 & 3 \end{pmatrix}$ is the inverse of $\begin{pmatrix} 7 & -8 & 5 \\ -4 & 5 & -3 \\ y & -1 & 1 \end{pmatrix}$ find x and y.

(III) Let $A = \begin{pmatrix} 0 & 1 \\ 1 & 1 \\ 0 & 1 \end{pmatrix}$ $and\ B = \begin{pmatrix} 1 & 0 & 0 \\ 1 & 1 & 0 \end{pmatrix}$ $be\ zero-one\ matrices. Find$

1) $A \odot B$

2) $A \vee B^t$

3) $A \wedge B^t$

Q#4:(I) How many vertices and how many edges do the following graphs have:

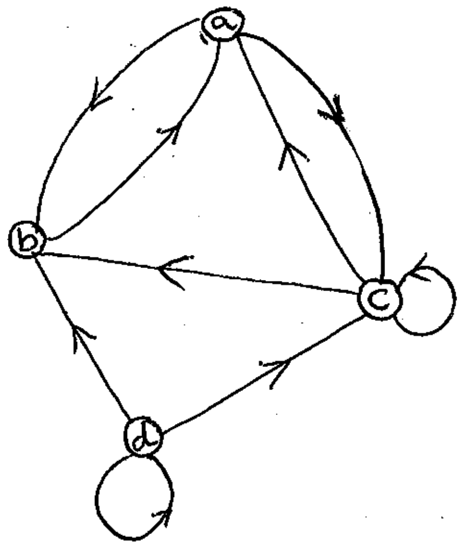
1) C_{200}

2) W_{32}

3) $K_{5,7}$

(II) Can a simple graph with 5 vertices each of degree 3 exist? Explain your answer.

(III) Find the in-degree and out-degree of each vertex for the given directed multigraph.



Q#5:(I) Determine whether the relation R on $\mathbb{Z}$ is reflexive, symmetric, antisymmetric, and/or transitive where

$$R = \{(a, b)\ , a < b + 2\}$$

(II) Prove that if m is odd and n is even then $(m^2 + n + 4)$ is odd.

Q#6:(I) If x is real number what is the value of the following:

1) $\lceil x \rceil - \lfloor x \rfloor =$

2) $\lceil x \rceil + \lfloor x \rfloor =$

(II) Find explicit formula for the sequence

$$2, \frac{-2}{3}, \frac{2}{9}, \frac{-2}{27}, \ldots\ldots\ldots\ldots\ldots\ldots$$

(III) Determine whether the statements below are true or false:

1) $\{\phi\} \subseteq P(\phi)$

2) $\{1,2\} \subset \{1,2\}$

3) If $A = \{\{0\},\{1\},\phi\}$ then $|P(A)| = 8$.

Best Wishes For All