Princess Sumaya University for Technology جامعة الأميرة سمية للتكنولوجيا

King Abdullah II School of Engineering

EE21221
Electric Circuits (1)
Section #1

Quiz #6
Thursday 13/1/2022

Name: ………………………………………………………………………

Q.1) **In the parallel circuit of Figure Q.1, find v(t) for t>0, assuming $v(0)=V_o=5$ V, $i(0)=I_o=0$ A, R=5 Ohms, L=1 H, and C=10 mF. [5-Points]**

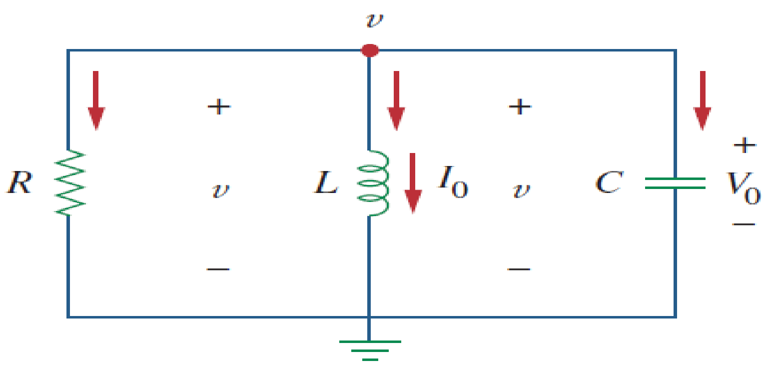


Figure Q.1

TABLE **9.1** Summary of Relevant Equations for Source-Free *RLC* Circuits

Type	Condition	Criteria	α	ω_0	Response
Parallel Series	Overdamped	$\alpha > \omega_0$	$\frac{1}{2RC}$ $\frac{R}{2L}$	$\frac{1}{\sqrt{LC}}$	$A_1e^{s_1t} + A_2e^{s_2t}$, where $s_{1,2} = -\alpha \pm \sqrt{\alpha^2 - \omega^2}$
Parallel Series	Critically damped	$\alpha = \omega_0$	$\frac{1}{2RC}$ $\frac{R}{2L}$	$\frac{1}{\sqrt{LC}}$	$e^{-\alpha t}(A_1t + A_2)$
Parallel Series	Underdamped	$\alpha < \omega_0$	$\frac{1}{2RC}$ $\frac{R}{2L}$	$\frac{1}{\sqrt{LC}}$	$e^{-\alpha t}(B_1 \cos\omega_d t + B_2 \sin\omega_d t)$, where $\omega_d = \sqrt{\omega_0^2 - \alpha^2}$

Solution:

When $R = 5\ \Omega$,

$$\alpha = \frac{1}{2RC} = \frac{1}{2 \times 5 \times 10 \times 10^{-3}} = 10$$

while $\omega_0 = 10$ remains the same. Since $\alpha = \omega_0 = 10$, the response is critically damped. Hence, $s_1 = s_2 = -10$, and

$$v(t) = (A_1 + A_2 t)e^{-10t} \qquad \mathbf{5}$$

To get A_1 and A_2, we apply the initial conditions

$$v(0) = 5 = A_1 \qquad \mathbf{6}$$

$$\frac{dv(0)}{dt} = -\frac{v(0) + Ri(0)}{RC} = -\frac{5 + 0}{5 \times 10 \times 10^{-3}} = -100$$

But differentiating Eq. 5 ,

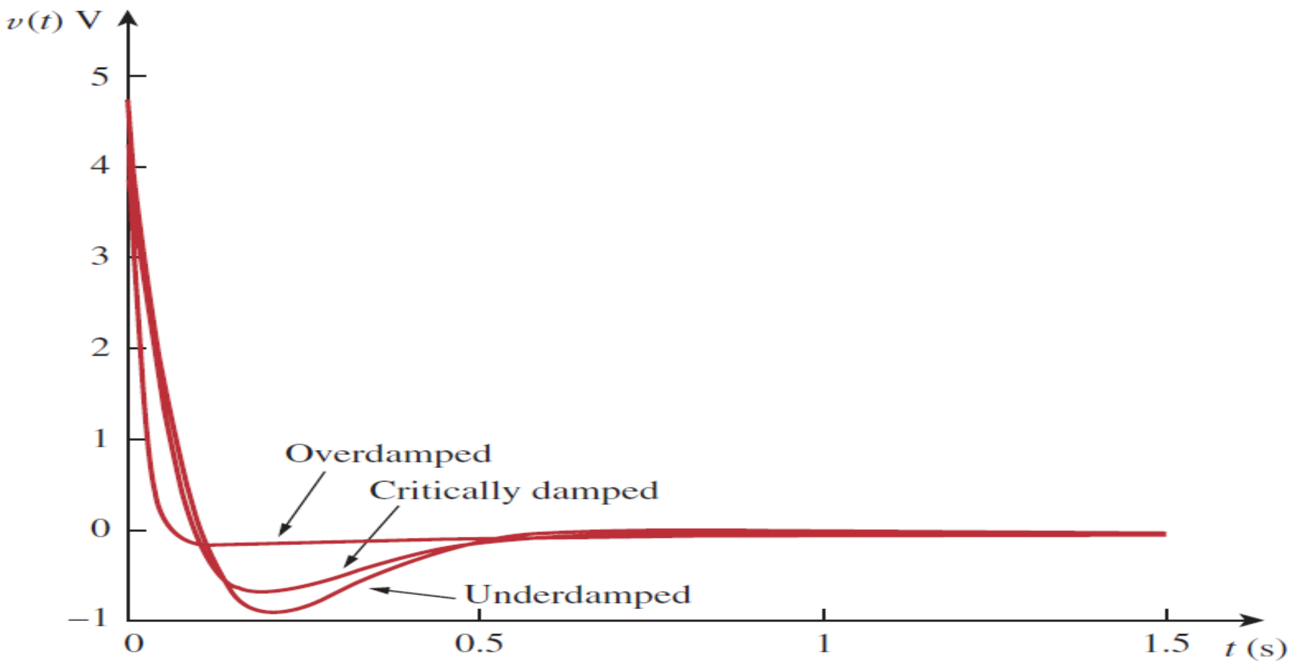
$$\frac{dv}{dt} = (-10A_1 - 10A_2 t + A_2)e^{-10t}$$

At $t = 0$,

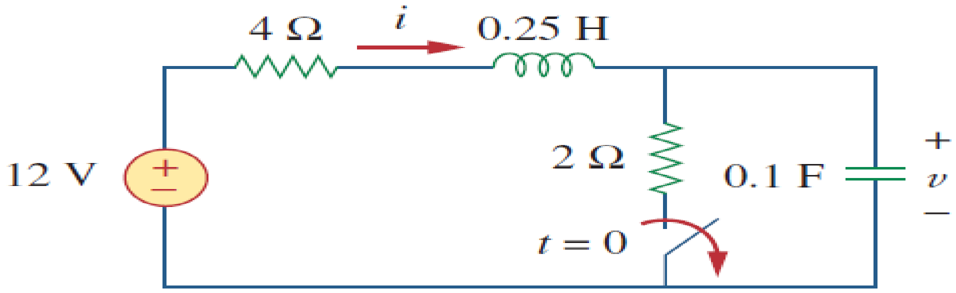
$$-100 = -10A_1 + A_2 \qquad \mathbf{7}$$

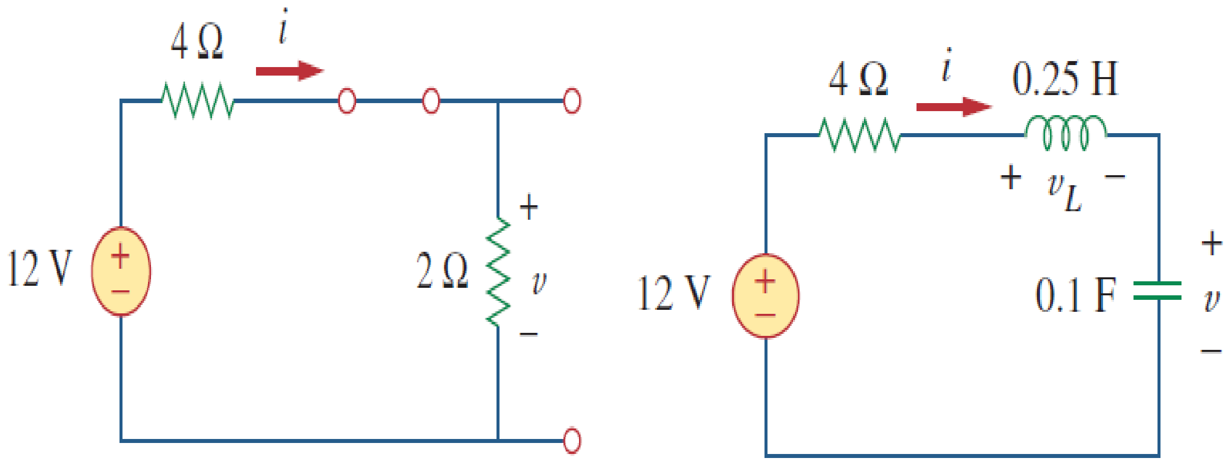
From Eqs. 6 and 7 , $A_1 = 5$ and $A_2 = -50$. Thus,

$$v(t) = (5 - 50t)e^{-10t}\ \text{V} \qquad \mathbf{8}$$



Q.2) The switch in Figure Q.2 has been closed for a long time. It is open at t=0. Find: i(0+), v(0+), i(∞), and v(∞). [5-Points]





(a) (b)

Figure Q.2

Equivalent circuit of that in Fig.2. for: (a) $t = 0^-$, (b) $t = 0^+$

Solution:

(a) If the switch is closed a long time before $t = 0$, it means that the circuit has reached dc steady state at $t = 0$. At dc steady state, the inductor acts like a short circuit, while the capacitor acts like an open circuit, so we have the circuit in Fig. 8.3(a) at $t = 0^-$. Thus,

$$i(0^-) = \frac{12}{4 + 2} = 2 \text{ A}, \qquad v(0^-) = 2i(0^-) = 4 \text{ V}$$

As the inductor current and the capacitor voltage cannot change abruptly,

$$i(0^+) = i(0^-) = 2 \text{ A}, \qquad v(0^+) = v(0^-) = 4 \text{ V}$$

(b) At $t = 0^+$, the switch is open; the equivalent circuit is as shown in Fig. 8.3(b). The same current flows through both the inductor and capacitor. Hence,

$$i_C(0^+) = i(0^+) = 2\text{ A}$$

Since $C\,dv/dt = i_C$, $dv/dt = i_C/C$, and

$$\frac{dv(0^+)}{dt} = \frac{i_C(0^+)}{C} = \frac{2}{0.1} = 20\text{ V/s}$$

Similarly, since $L\,di/dt = v_L$, $di/dt = v_L/L$. We now obtain v_L by applying KVL to the loop in Fig. 8.3(b). The result is

$$-12 + 4i(0^+) + v_L(0^+) + v(0^+) = 0$$

or

$$v_L(0^+) = 12 - 8 - 4 = 0$$

Thus,

$$\frac{di(0^+)}{dt} = \frac{v_L(0^+)}{L} = \frac{0}{0.25} = 0\text{ A/s}$$

(c) For $t > 0$, the circuit undergoes transience. But as $t \to \infty$, the circuit reaches steady state again. The inductor acts like a short circuit and the capacitor like an open circuit, so that the circuit in Fig. 8.3(b) becomes that shown in Fig. 8.3(c), from which we have

$$i(\infty) = 0\text{ A}, \qquad v(\infty) = 12\text{ V}$$