Princess Sumaya University for Technology
جامعة الأميرة سمية للتكنولوجيا

King Abdullah II School of Engineering

EE21221
Electric Circuits (1)
Section #3

Quiz #6
Wednesday 12/1/2022

Name: ………………………………………………………………..

Q.1) In the parallel circuit of Figure Q.1, find v(t) for t>0, assuming $v(0)=V_o=5$ V, $i(0)=I_o=0$ A, R=1.923 Ohms, L=1 H, and C=10 mF. [4-Points]

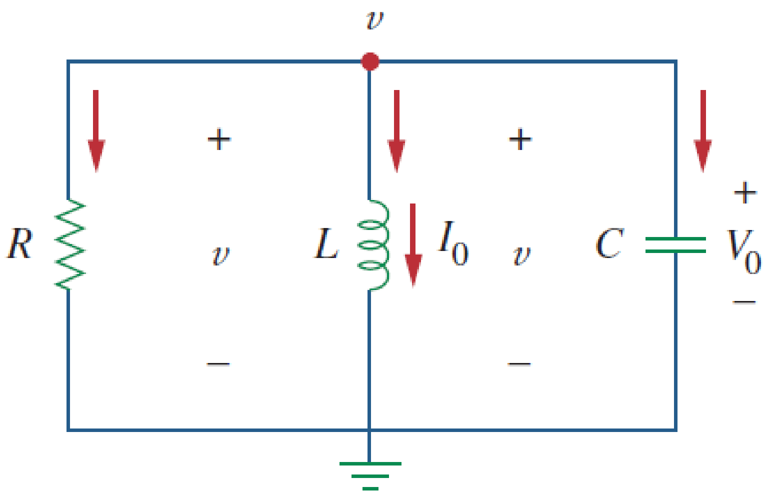


Figure Q.1

Solution:

If $R = 1.923\ \Omega$,

$$\alpha = \frac{1}{2RC} = \frac{1}{2 \times 1.923 \times 10 \times 10^{-3}} = 26$$

$$\omega_0 = \frac{1}{\sqrt{LC}} = \frac{1}{\sqrt{1 \times 10 \times 10^{-3}}} = 10$$

Since $\alpha > \omega_0$ in this case, the response is overdamped. The roots of the characteristic equation are

$$s_{1,2} = -\alpha \pm \sqrt{\alpha^2 - \omega_0^2} = -2, -50$$

and the corresponding response is

$$v(t) = A_1 e^{-2t} + A_2 e^{-50t} \qquad 1$$

We now apply the initial conditions to get A_1 and A_2.

$$v(0) = 5 = A_1 + A_2 \qquad 2$$

$$\frac{dv(0)}{dt} = -\frac{v(0) + Ri(0)}{RC} = -\frac{5 + 0}{1.923 \times 10 \times 10^{-3}} = -260$$

But differentiating Eq.1

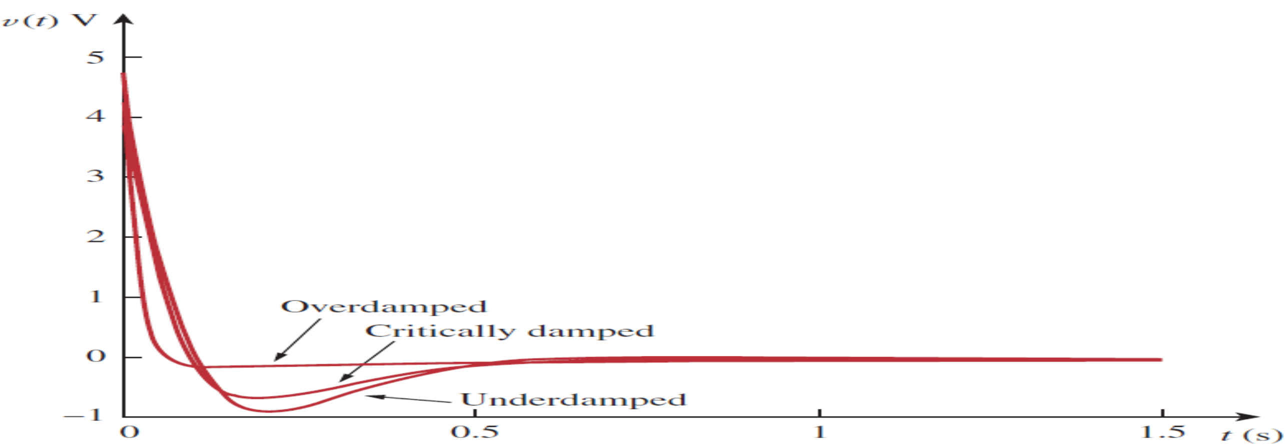
$$\frac{dv}{dt} = -2A_1 e^{-2t} - 50A_2 e^{-50t}$$

At $t = 0$,

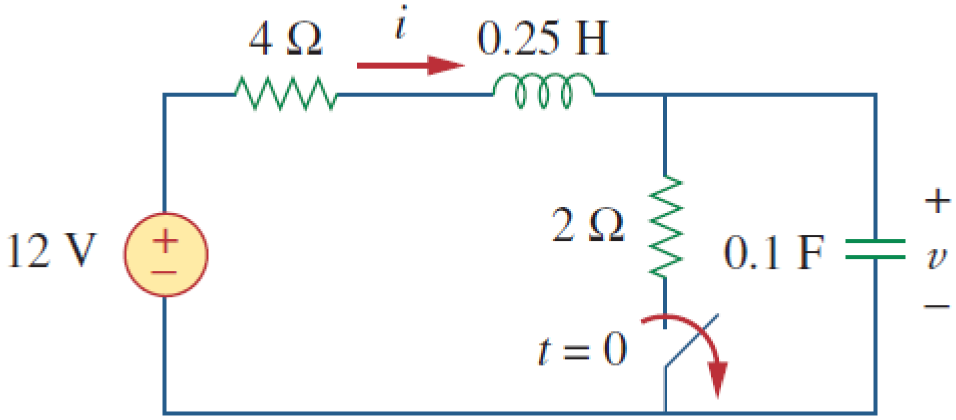
$$-260 = -2A_1 - 50A_2 \qquad 3$$

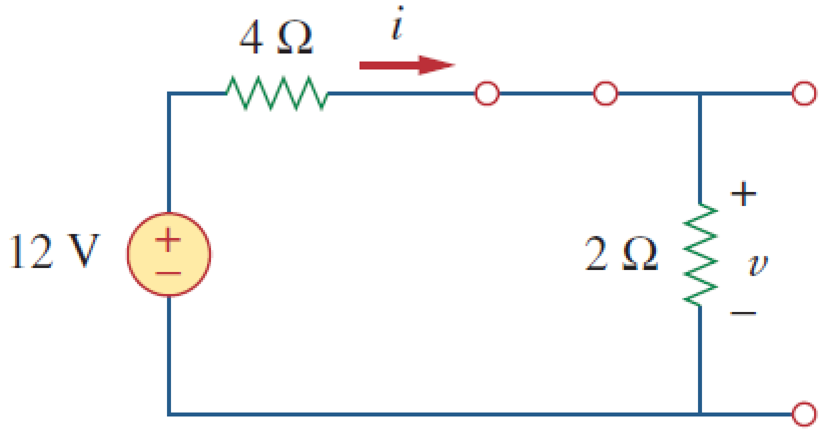
From Eqs. .2 and .3 , we obtain $A_1 = -0.2083$ and $A_2 = 5.208$. Substituting A_1 and A_2 in Eq. 1 yields

$$v(t) = -0.2083e^{-2t} + 5.208e^{-50t} \qquad 4$$

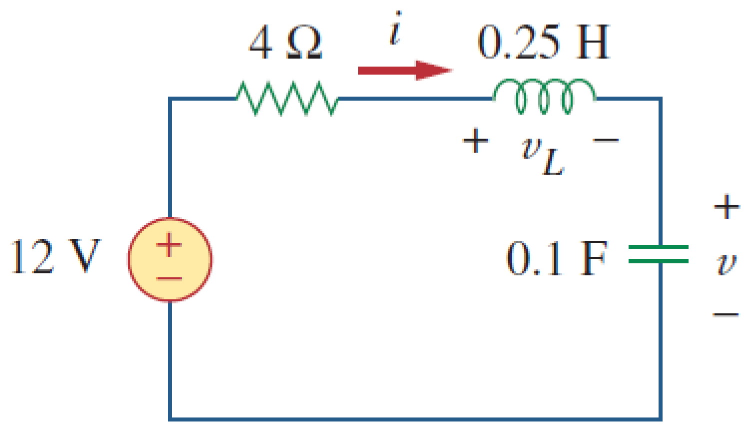


Q.2) The switch in Figure Q.2 has been closed for a long time. It is open at t=0. Find:
(a) i(0+), v(0+), i(∞), and v(∞). [4-Points]





(a)



(b)

Figure Q.2

Equivalent circuit of that in Fig. 2. for: (a) $t = 0^-$, (b) $t = 0^+$

Solution:

(a) If the switch is closed a long time before $t = 0$, it means that the circuit has reached dc steady state at $t = 0$. At dc steady state, the inductor acts like a short circuit, while the capacitor acts like an open circuit, so we have the circuit in Fig. 8.3(a) at $t = 0^-$. Thus,

$$i(0^-) = \frac{12}{4+2} = 2\text{ A}, \qquad v(0^-) = 2i(0^-) = 4\text{ V}$$

As the inductor current and the capacitor voltage cannot change abruptly,

$$i(0^+) = i(0^-) = 2\text{ A}, \qquad v(0^+) = v(0^-) = 4\text{ V}$$

(b) At $t = 0^+$, the switch is open; the equivalent circuit is as shown in Fig. 8.3(b). The same current flows through both the inductor and capacitor. Hence,

$$i_C(0^+) = i(0^+) = 2\text{ A}$$

Since $C\,dv/dt = i_C$, $dv/dt = i_C/C$, and

$$\frac{dv(0^+)}{dt} = \frac{i_C(0^+)}{C} = \frac{2}{0.1} = 20\text{ V/s}$$

Similarly, since $L\,di/dt = v_L$, $di/dt = v_L/L$. We now obtain v_L by applying KVL to the loop in Fig. 8.3(b). The result is

$$-12 + 4i(0^+) + v_L(0^+) + v(0^+) = 0$$

or

$$v_L(0^+) = 12 - 8 - 4 = 0$$

Thus,

$$\frac{di(0^+)}{dt} = \frac{v_L(0^+)}{L} = \frac{0}{0.25} = 0\text{ A/s}$$

(c) For $t > 0$, the circuit undergoes transience. But as $t \to \infty$, the circuit reaches steady state again. The inductor acts like a short circuit and the capacitor like an open circuit, so that the circuit in Fig. 8.3(b) becomes that shown in Fig. 8.3(c), from which we have

$$i(\infty) = 0\text{ A}, \qquad v(\infty) = 12\text{ V}$$

TABLE 9.1 Summary of Relevant Equations for Source-Free *RLC* Circuits

Type	Condition	Criteria	α	ω_0	Response
Parallel Series	Overdamped	$\alpha > \omega_0$	$\frac{1}{2RC}$ $\frac{R}{2L}$	$\frac{1}{\sqrt{LC}}$	$A_1 e^{s_1 t} + A_2 e^{s_2 t}$, where $s_{1,2} = -\alpha \pm \sqrt{\alpha^2 - \omega_0^2}$
Parallel Series	Critically damped	$\alpha = \omega_0$	$\frac{1}{2RC}$ $\frac{R}{2L}$	$\frac{1}{\sqrt{LC}}$	$e^{-\alpha t}(A_1 t + A_2)$
Parallel Series	Underdamped	$\alpha < \omega_0$	$\frac{1}{2RC}$ $\frac{R}{2L}$	$\frac{1}{\sqrt{LC}}$	$e^{-\alpha t}(B_1 \cos \omega_d t + B_2 \sin \omega_d t)$, where $\omega_d = \sqrt{\omega_0^2 - \alpha^2}$