Name: …………………………………………………………………….

Q.1) In the parallel circuit of Figure Q.1, find v(t) for t>0, assuming v(0)=V$_o$=5 V, i(0)=I$_o$=0 A, R=6.25 Ohms, L=1 H, and C=10 mF. **[5-Points]**

Figure Q.1

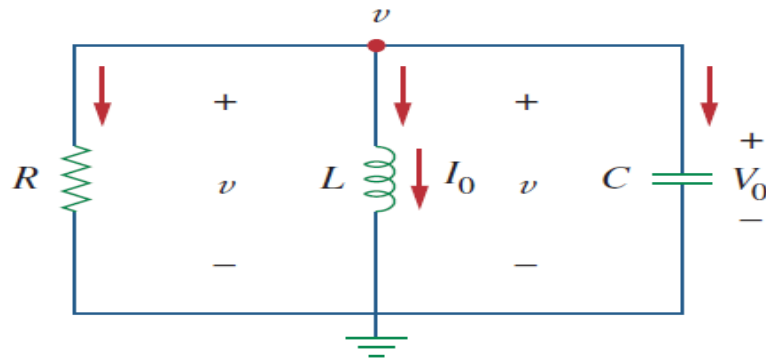TABLE **9.1** Summary of Relevant Equations for Source-Free *RLC* Circuits

Type	Condition	Criteria	α	ω_0	Response
Parallel	Overdamped	$\alpha > \omega_0$	$\dfrac{1}{2RC}$	$\dfrac{1}{\sqrt{LC}}$	$A_1 e^{s_1 t} + A_2 e^{s_2 t}$, where
Series			$\dfrac{R}{2L}$		$s_{1,2} = -\alpha \pm \sqrt{\alpha^2 - \omega^2}$
Parallel	Critically damped	$\alpha = \omega_0$	$\dfrac{1}{2RC}$	$\dfrac{1}{\sqrt{LC}}$	$e^{-\alpha t}(A_1 t + A_2)$
Series			$\dfrac{R}{2L}$		
Parallel	Underdamped	$\alpha < \omega_0$	$\dfrac{1}{2RC}$	$\dfrac{1}{\sqrt{LC}}$	$e^{-\alpha t}(B_1 \cos \omega_d t + B_2 \sin \omega_d t)$,
Series			$\dfrac{R}{2L}$		where $\omega_d = \sqrt{\omega_0^2 - \alpha^2}$

Solution:

When $R = 6.25 \, \Omega$,

$$\alpha = \frac{1}{2RC} = \frac{1}{2 \times 6.25 \times 10 \times 10^{-3}} = 8$$

while $\omega_0 = 10$ remains the same. As $\alpha < \omega_0$ in this case, the response is underdamped. The roots of the characteristic equation are

$$s_{1,2} = -\alpha \pm \sqrt{\alpha^2 - \omega_0^2} = -8 \pm j6$$

Hence,

$$v(t) = (A_1 \cos 6t + A_2 \sin 6t)e^{-8t} \qquad \qquad 9$$

We now obtain A_1 and A_2, as

$$v(0) = 5 = A_1 \qquad \qquad 10$$

$$\frac{dv(0)}{dt} = -\frac{v(0) + Ri(0)}{RC} = -\frac{5 + 0}{6.25 \times 10 \times 10^{-3}} = -80$$

But differentiating Eq. 9 ,

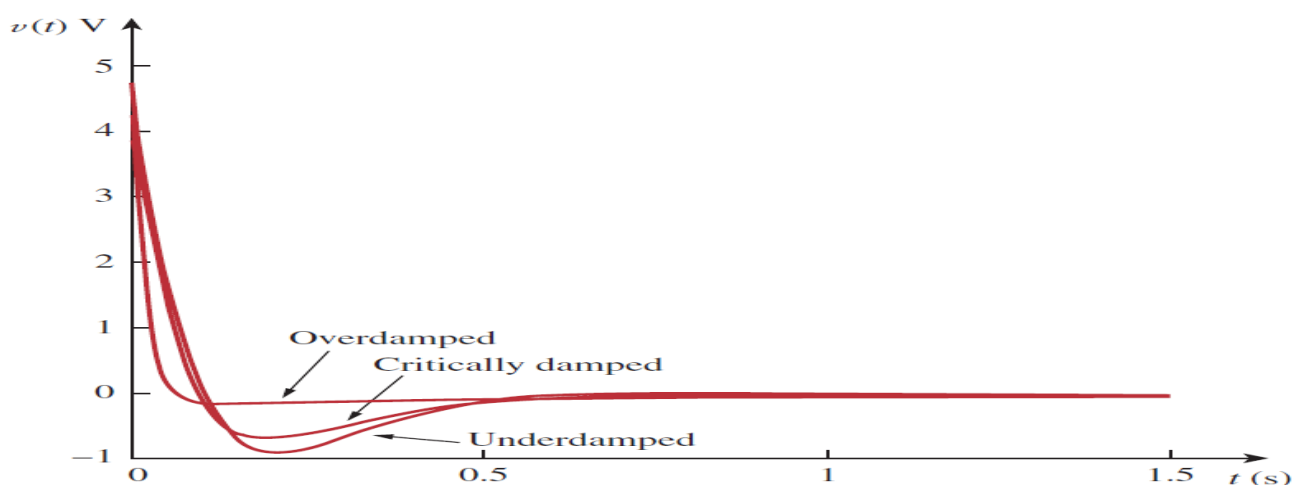$$\frac{dv}{dt} = (-8A_1 \cos 6t - 8A_2 \sin 6t - 6A_1 \sin 6t + 6A_2 \cos 6t)e^{-8t}$$

At $t = 0$,

$$-80 = -8A_1 + 6A_2 \qquad \qquad 11$$

From Eqs. 10 and 11 , $A_1 = 5$ and $A_2 = -6.667$. Thus,

$$v(t) = (5 \cos 6t - 6.667 \sin 6t)e^{-8t} \qquad \qquad 12$$

Notice that by increasing the value of R, the degree of damping decreases and the responses differ.

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Q.2) The switch in Figure Q.2 has been closed for a long time. It is open at t=0. Find:
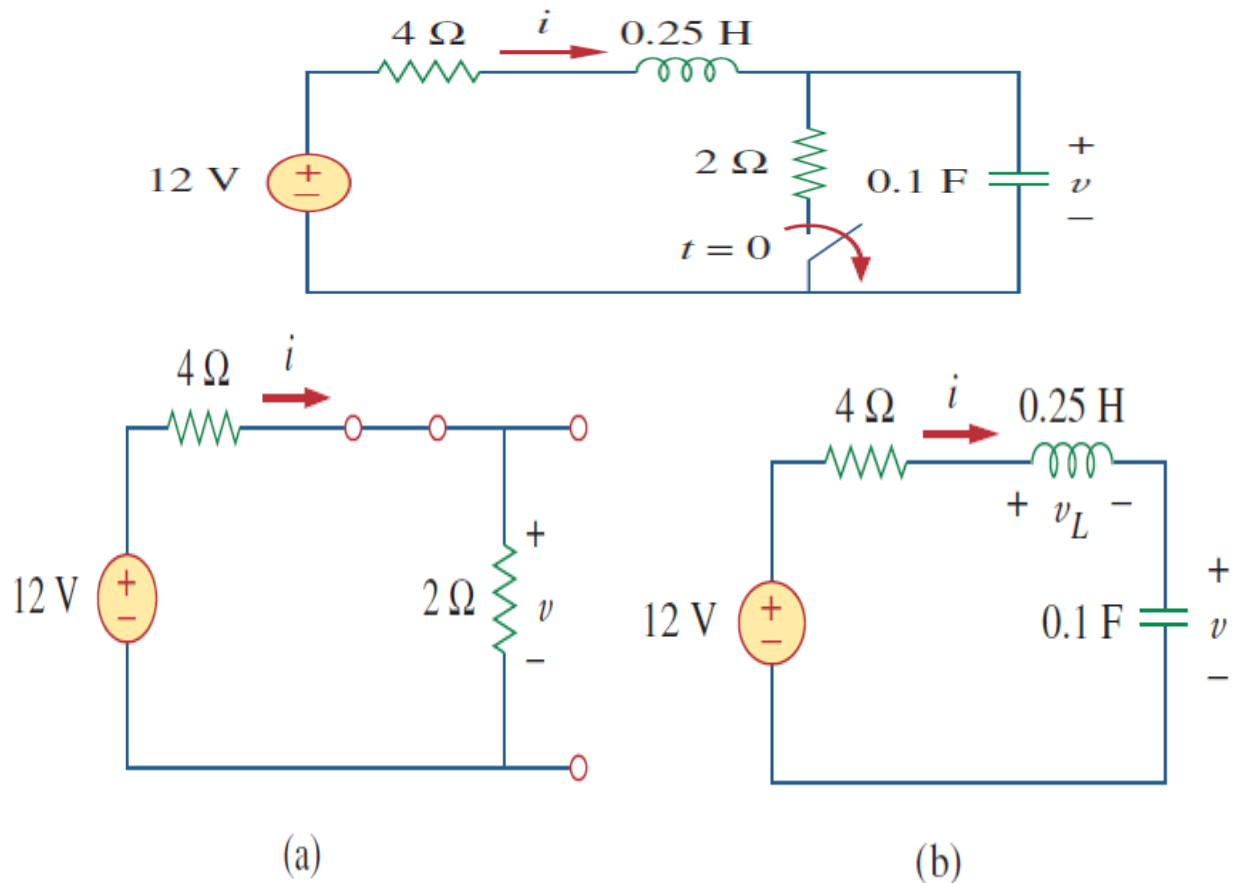i(0⁺), v(0⁺), di(0⁺)/dt, dv(0⁺)/dt, i(∞), and v(∞). [5-Points]

Figure Q.2

Equivalent circuit of that in Fig. 2. for: (a) $t = 0^-$, (b) $t = 0^+$

Solution:

(a) If the switch is closed a long time before $t = 0$, it means that the circuit has reached dc steady state at $t = 0$. At dc steady state, the inductor acts like a short circuit, while the capacitor acts like an open circuit, so we have the circuit in Fig. 8.3(a) at $t = 0^-$. Thus,

$$i(0^-) = \frac{12}{4 + 2} = 2 \text{ A}, \qquad v(0^-) = 2i(0^-) = 4 \text{ V}$$

As the inductor current and the capacitor voltage cannot change abruptly,

$$i(0^+) = i(0^-) = 2 \text{ A}, \qquad v(0^+) = v(0^-) = 4 \text{ V}$$

(b) At $t = 0^+$, the switch is open; the equivalent circuit is as shown in Fig. 8.3(b). The same current flows through both the inductor and capacitor. Hence,

$$i_C(0^+) = i(0^+) = 2 \text{ A}$$

Since $C \, dv/dt = i_C$, $dv/dt = i_C/C$, and

$$\frac{dv(0^+)}{dt} = \frac{i_C(0^+)}{C} = \frac{2}{0.1} = 20 \text{ V/s}$$

Similarly, since $L \, di/dt = v_L$, $di/dt = v_L/L$. We now obtain v_L by applying KVL to the loop in Fig. 8.3(b). The result is

$$-12 + 4i(0^+) + v_L(0^+) + v(0^+) = 0$$

or

$$v_L(0^+) = 12 - 8 - 4 = 0$$

Thus,

$$\frac{di(0^+)}{dt} = \frac{v_L(0^+)}{L} = \frac{0}{0.25} = 0 \text{ A/s}$$

(c) For $t > 0$, the circuit undergoes transience. But as $t \to \infty$, the circuit reaches steady state again. The inductor acts like a short circuit and the capacitor like an open circuit, so that the circuit in Fig. 8.3(b) becomes that shown in Fig. 8.3(c), from which we have

$$i(\infty) = 0 \text{ A}, \qquad v(\infty) = 12 \text{ V}$$