King Abdullah II School of Engineering

EE21221
Electric Circuits (1)
Section #3

Quiz #4
Wednesday 29/12/2021

Name: ……………………………………………………………….

Q.1) Sketch the voltage which develops across the terminals of a 2.5 F capacitor in response to the current waveforms that is shown in Figure Q.1. [3-Points]

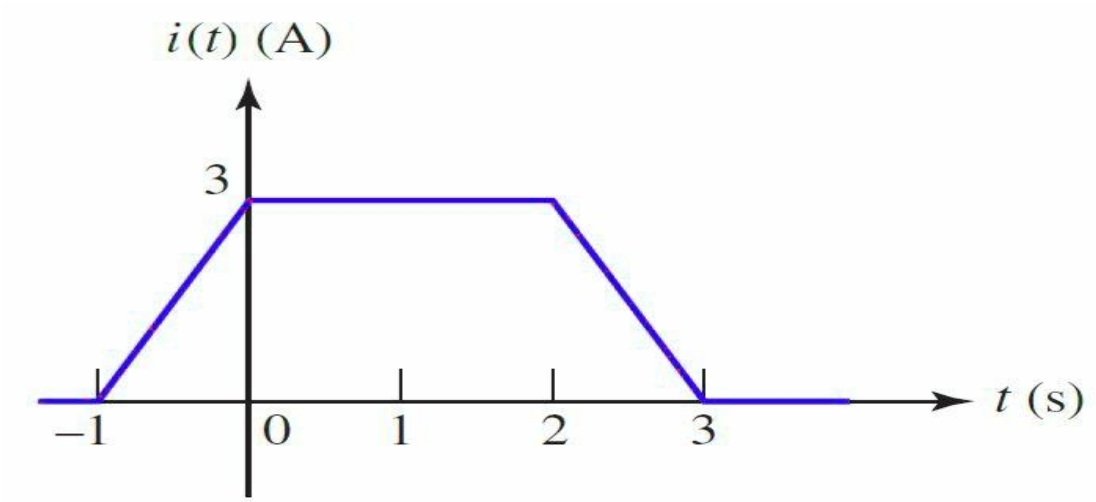
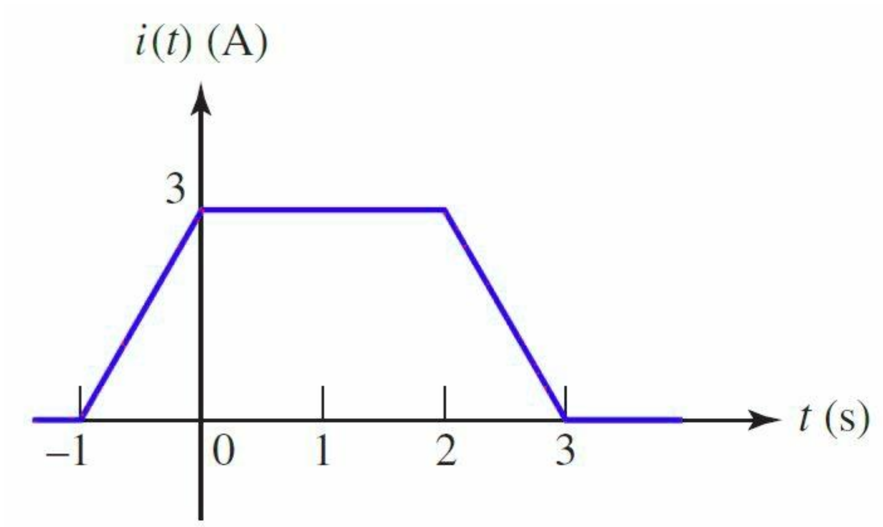
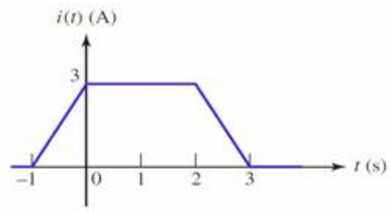


Figure Q.1

Solution:





For $-\infty \le t \le -1$

$$i(t) = 0, -\infty \le t \le -1$$
$$v(t_o) = v(-\infty) = 0$$
$$v(t) = \frac{1}{2.5}\int_{-\infty}^{t} 0.\,dt + v(-\infty)$$
$$v(t) = 0, -\infty \le t \le -1$$

For $-1 \le t \le 0$

$$i(t) = 3t + 3, -1 \le t \le 0$$
$$v = \frac{1}{2.5}\int_{-1}^{t} 3t + 3.\,dt + v(-1)$$
$$= \frac{1}{2.5}(1.5\,t^2 + 3t)|_{-1}^{t} + v(-1)$$
$$v(-1) = 0$$
$$v = \frac{1}{2.5}(1.5\,t^2 + 3t + 1.5)$$
$$v(t) = 0.6\,t^2 + 1.2t + 0.6\,, -1 \le t \le 0$$

For $0 \le t \le 2$

$$i(t) = 3\,, 0 \le t \le 2$$
$$v = \frac{1}{2.5}\int_{0}^{t} 3.\,dt + v(0)$$
$$= \frac{3}{2.5}t|_0^t + v(0)$$
$$v(0) = 0.6$$
$$v = \frac{3}{2.5}t + 0.6$$
$$v(t) = 1.2t + 0.6\,, 0 \le t \le 2$$

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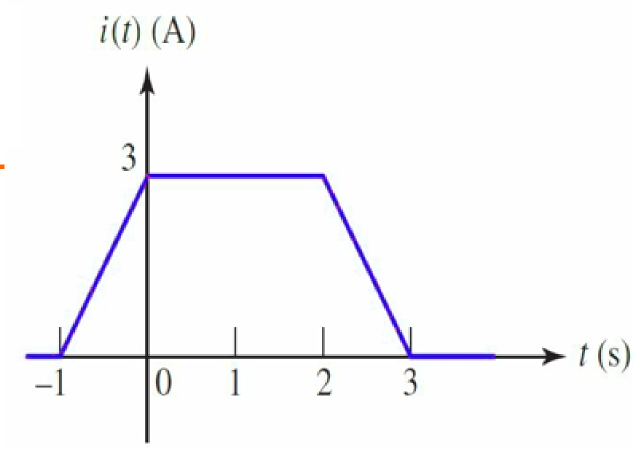
For $t \ge 3$

$$v(t) = \frac{1}{2.5}\int_{3}^{t} 0.\,dt + v(3)$$
$$v(3) = 3.6$$
$$v(t) = 3.6\,, t \ge 3$$



$v(t) = 0, -\infty \le t \le -1$

$v(t) = 0.6\,t^2 + 1.2t + 0.6\,, -1 \le t \le 0$

$v(t) = 1.2t + 0.6\,, 0 \le t \le 2$

$v(t) = 3.6\,, t \ge 3$

For $2 \le t \le 3$

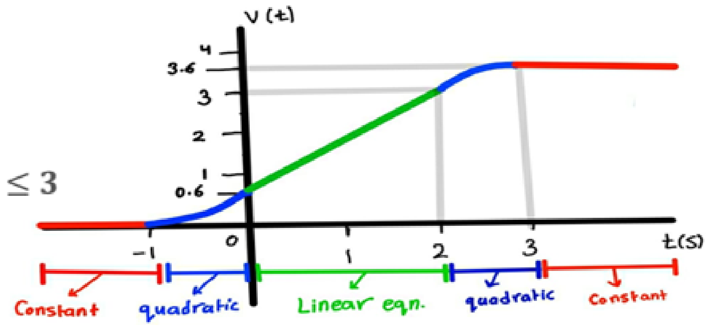
$$v(t) = \frac{1}{2.5}\int_2^t 9 - 3t\,.dt + v(2)$$

$$= \frac{1}{2.5}\left(9t - 1.5t^2\right)\Big|_2^t + v(2)$$

$v(2) = 3$

$v(t) = -0.6t^2 + 3.6t - 1.8\ , 2 \le t \le 3$

$$V(t) = \begin{cases} 0 & , t \leqslant -1 \\ 0.6t^2 + 1.2t + 0.6 & , -1 \leqslant t \leqslant 0 \\ 1.2t + 0.6 & , 0 \leqslant t \leqslant 2 \\ -0.6t^2 + 3.6t - 1.8 & , 2 \leqslant t \leqslant 3 \\ 3.6 & , t \geqslant 3 \end{cases}$$



Q.2) Obtain the equivalent resistance R_{ab} in the circuit shown in Figure Q.2 then use it to find i. [4-Points]

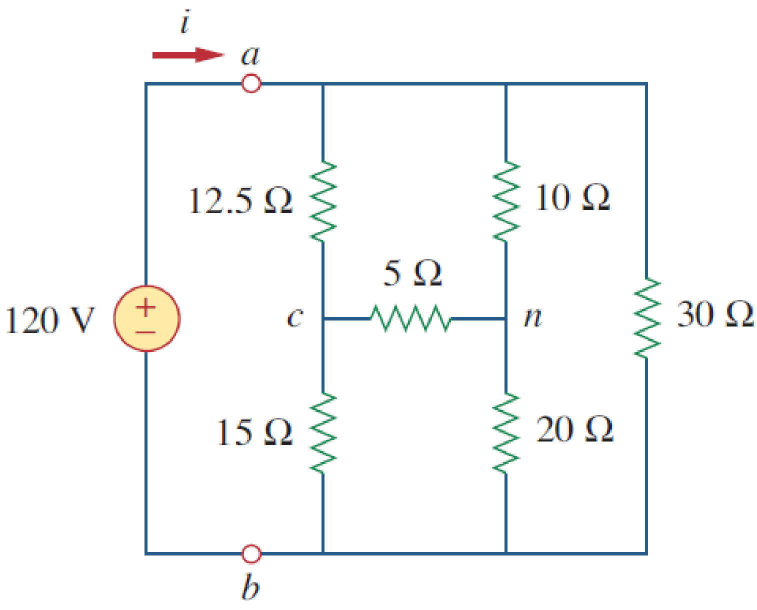
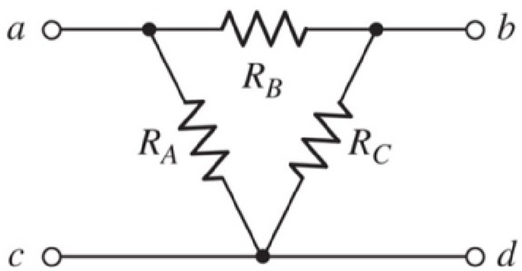


Figure Q.2

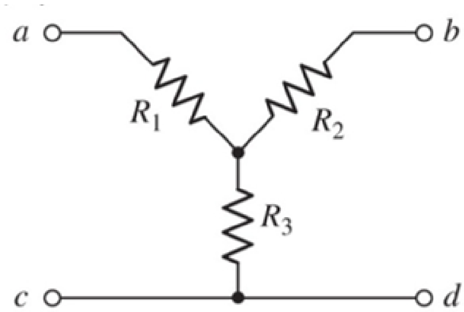


this Δ is equivalent to the Y if

$$R_A = \frac{R_1R_2 + R_2R_3 + R_3R_1}{R_2}$$

$$R_B = \frac{R_1R_2 + R_2R_3 + R_3R_1}{R_3}$$

$$R_C = \frac{R_1R_2 + R_2R_3 + R_3R_1}{R_1}$$



this Y is equivalent to the Δ if

$$R_1 = \frac{R_AR_B}{R_A + R_B + R_C}$$

$$R_2 = \frac{R_BR_C}{R_A + R_B + R_C}$$

$$R_3 = \frac{R_CR_A}{R_A + R_B + R_C}$$

Solution:

i=

Two Methods:

First one:

$$R_1 = 10\ \Omega, \qquad R_2 = 20\ \Omega, \qquad R_3 = 5\ \Omega$$

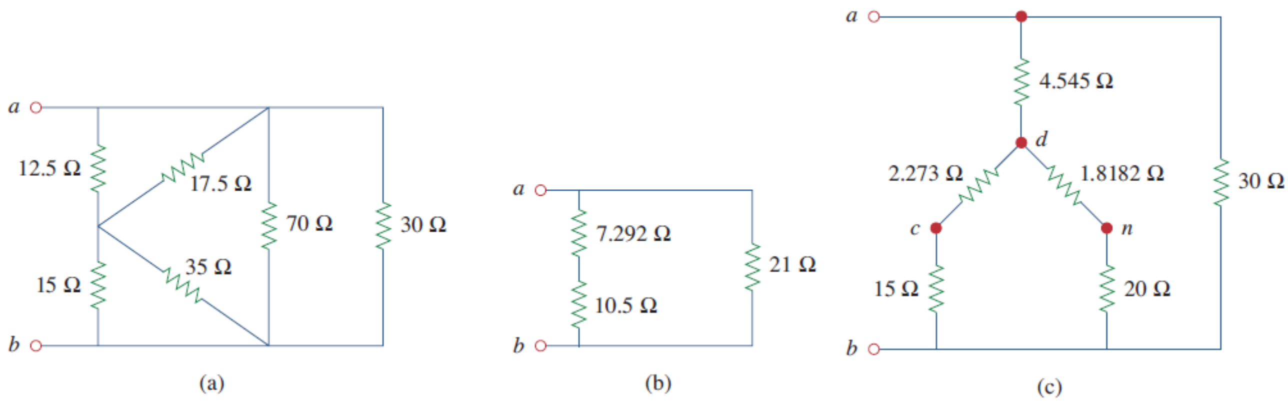
Thus from Eqs. (2.53) to (2.55) we have

$$R_a = \frac{R_1R_2 + R_2R_3 + R_3R_1}{R_1} = \frac{10 \times 20 + 20 \times 5 + 5 \times 10}{10}$$

$$= \frac{350}{10} = 35\ \Omega$$

$$R_b = \frac{R_1R_2 + R_2R_3 + R_3R_1}{R_2} = \frac{350}{20} = 17.5\ \Omega$$

$$R_c = \frac{R_1R_2 + R_2R_3 + R_3R_1}{R_3} = \frac{350}{5} = 70\ \Omega$$



With the Y converted to Δ, the equivalent circuit (with the voltage source removed for now) is shown in Fig. 2.53(a). Combining the three pairs of resistors in parallel, we obtain

$$70 \| 30 = \frac{70 \times 30}{70 + 30} = 21\ \Omega$$

$$12.5 \| 17.5 = \frac{12.5 \times 17.5}{12.5 + 17.5} = 7.292\ \Omega$$

$$15 \| 35 = \frac{15 \times 35}{15 + 35} = 10.5\ \Omega$$

so that the equivalent circuit is shown in Fig. 2.53(b). Hence, we find

$$R_{ab} = (7.292 + 10.5) \| 21 = \frac{17.792 \times 21}{17.792 + 21} = \mathbf{9.632\ \Omega}$$

Then

$$i = \frac{v_s}{R_{ab}} = \frac{120}{9.632} = \mathbf{12.458\ A}$$

Second Method:

Evaluate. Now we must determine if the answer is correct and then evaluate the final solution.

It is relatively easy to check the answer; we do this by solving the problem starting with a delta-wye transformation. Let us transform the delta, *can*, into a wye.

Let $R_c = 10\ \Omega$, $R_a = 5\ \Omega$, and $R_n = 12.5\ \Omega$. This will lead to (let d represent the middle of the wye):

$$R_{ad} = \frac{R_c R_n}{R_a + R_c + R_n} = \frac{10 \times 12.5}{5 + 10 + 12.5} = 4.545\ \Omega$$

$$R_{cd} = \frac{R_a R_n}{27.5} = \frac{5 \times 12.5}{27.5} = 2.273\ \Omega$$

$$R_{nd} = \frac{R_a R_c}{27.5} = \frac{5 \times 10}{27.5} = 1.8182\ \Omega$$

This now leads to the circuit shown in Figure 2.53(c). Looking at the resistance between d and b, we have two series combination in parallel, giving us

$$R_{db} = \frac{(2.273 + 15)(1.8182 + 20)}{2.273 + 15 + 1.8182 + 20} = \frac{376.9}{39.09} = 9.642\ \Omega$$

This is in series with the 4.545-Ω resistor, both of which are in parallel with the 30-Ω resistor. This then gives us the equivalent resistance of the circuit.

$$R_{ab} = \frac{(9.642 + 4.545)30}{9.642 + 4.545 + 30} = \frac{425.6}{44.19} = \mathbf{9.631\ \Omega}$$

This now leads to

$$i = \frac{v_s}{R_{ab}} = \frac{120}{9.631} = \mathbf{12.46\ A}$$

We note that using two variations on the wye-delta transformation leads to the same results. This represents a very good check.

Q.3) Obtain the energy stored in the 4 mF capacitor that is shown in Fig. Q.3 under dc conditions. [3-Points]

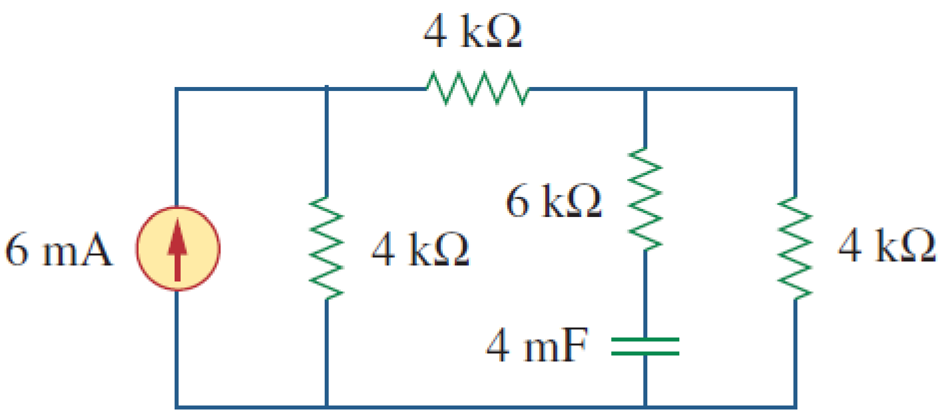


Figure Q.3

Solution:

W_{4mF}= $\boxed{}$

i_{4k}=(6mA*4K)/12K=2mA

Vc=2mA*4K=8V

$W=0.5CV^2$=0.5*4m*8*8=0.128 J