Spring Semester: 2024 -2025

Calculus 3 (20231)

Princess Sumaya University for Technology جامعة الأميرة سمية للتكنولوجيا

Midterm-exam
Duration (60) minutes
Grade /30
Date: 17-4-2025

King Abdullah II school of Engineering – Department of Basic Sciences

Name: -- **ID** -------------------------

Q.1: (10 points)

Part (A) Determine whether each statement is TRUE or FALSE:

i- *For any two vectors* u *and* v *in* $\mathbb{R}^3$, $|u \times v| = |v \times u|$. (T)

ii- $If\ u.v = 0\ and\ u \times v = \mathbf{0}\ then\ u = \mathbf{0}\ or\ v = \mathbf{0}$. (T)

Part(B) Fill in the blanks with the CORRECT answers ONLY:

i) $If\ a.b = \sqrt{2}\ and\ a \times b =< 1,-1,0 >$ *then the angle between a and b is:* ($\frac{\pi}{4}$ or 45°)

ii) *A vector that has same direction as* $\sqrt{7}i + 2j - 5k$ *of length* 12 *is* ($2\sqrt{7}i + 4j - 10k$)

iii) *The distance from the point* $P(-5,2,-3)$ *to the* $z-axis$ *is:* ($\sqrt{29}$)

iv) *Let* $f(x,y,z) = \ln\left(z - \sqrt{x^2+y^2}\right)$, *and* $f(2,0,k) = 0$ *then* $k =$ (3)

Q.2: (3 points)

Showing each step, calculate: $\underline{[(i-j) \times (k-i)]}.(i+j)$

$$\begin{vmatrix} i & j & k \\ 1 & -1 & 0 \\ -1 & 0 & 1 \end{vmatrix} = -i - j - k$$

$(-i-j-k).(i+j)$

$\langle -1,-1,-1 \rangle . \langle 1,1,0 \rangle = -1 + -1 + 0 = -2$

Q.3: **(5 points)**

Part (A)

Show that $\lim_{(x,y)\to(0,0)} \frac{xy+y^3}{x^2+y^2}$ *does not exit.* ***(DO NOT USE POLAR COORDNITES)***

Along $\boxed{y=x}$

$$\lim_{x\to 0} \frac{x^2+x^3}{x^2+x^2} = \lim_{x\to 0} \frac{x^2(1+x)}{2x^2}$$

$$= \lim_{x\to 0} \tfrac{1}{2}(1+x)$$

$$= \frac{1}{2}$$

Along $\boxed{y=x^2}$

$$\lim_{x\to 0} \frac{x^3+x^6}{x^2+x^4}$$

$$= \lim_{x\to 0} \frac{x^3(1+x^3)}{x^2(1+x^2)}$$

$$= \lim_{x\to 0} x\left(\frac{1+x^3}{1+x^2}\right) = 0$$

Different Paths lead to different limiting values, the given limit does not exist.

Part(B)

Use ***polar coordinates*** *to determine whether the following limit exits or not:*

$\lim_{(x,y)\to(0,0)} \frac{4xy}{-x^2-y^2}$ $\quad x = r\cos\theta \quad y = r\sin\theta \quad x^2+y^2=r^2$

$$\lim_{(r,\theta)\to(0,\theta)} \frac{4r^2\cos\theta\sin\theta}{-r^2} = \lim_{(r,\theta)\to(0,\theta)} -4\sin\theta\cos\theta$$

Does not exist as the limit depends on θ

Q.4: (6 points)

Part(A) *Use implicit differentiation to find* $\frac{\partial z}{\partial x}$:

$e^{3z} = x^2 y \ln z$

$$3e^{3z} z_x = 2xy\ln z + x^2 y \frac{z_x}{z}$$

$$3ze^{3z} z_x = 2xyz\ln z + x^2 y z_x$$

$$z_x = \frac{2xyz\ln z}{3ze^{3z} - x^2 y}$$

Part(B): *Given that* $z = -\sqrt{x^2 + y^2}$

i – Identify the surface $z = -\sqrt{x^2 + y^2}$

ii – Find the equation of the tangent plane to z at $(1,0,-1)$

i- The lower part of an open cone.

ii- $z_x = -\frac{1}{2}\frac{(2x)}{\sqrt{x^2+y^2}} \rightarrow z_x = -1$

$z_y = -\frac{1}{2}\frac{(2y)}{\sqrt{x^2+y^2}} \rightarrow z_y = 0$

$z+1 = -1(x-1) + 0$

$z = -x$

Q.5 (6 points)

Part (A)

Given the points $A(5,1,-3), B(0,2,6),$ *and* $C(4,4,-2)$.

i)Find the point D which is the midpoint of the line segment from B to C.

ii)Find the parametric equations of the line L which passes through the point A and the point D.

i) Midpoint of BC $= \left(\frac{0+4}{2}, \frac{2+4}{2}, \frac{6+-2}{2}\right) = (2,3,2) \leftarrow D$

ii) $\overrightarrow{AD} = -3, 2, 5$

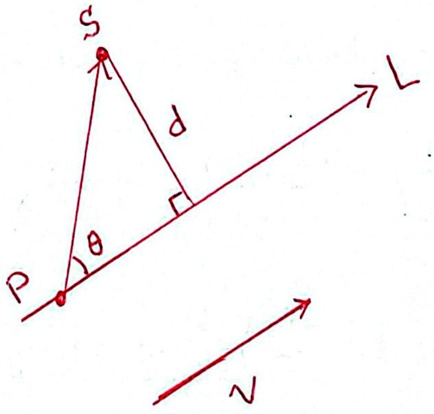
$L:\ x = 5 - 3t$

$y = 1 + 2t$

$z = -3 + 5t$

Part(B)

Given that the line L passes through the point P and parallel to the vector $v = < a, b, c >$. *Show that the distance (d) from the point S to the line L in the space is given by the formula:* $d = \frac{|\overrightarrow{PS} \times \vec{v}|}{|\vec{v}|}$



- $\sin\theta = \frac{d}{|\overrightarrow{PS}|}$

- $|\overrightarrow{PS} \times \vec{v}| = |\overrightarrow{PS}||\vec{v}| \sin\theta$

- $\sin\theta = \frac{|\overrightarrow{PS} \times \vec{v}|}{|\overrightarrow{PS}||\vec{v}|}$

$$\frac{d}{|\overrightarrow{PS}|} = \frac{|\overrightarrow{PS} \times \vec{v}|}{|\overrightarrow{PS}||\vec{v}|}$$

$$d = \frac{|\overrightarrow{PS} \times \vec{v}|}{|\vec{v}|}$$

END

GOOD LUCK