1

Spring Semester: 2024 -2025

Calculus 3 (20231)

Princess Sumaya University for Technology
جامعة الأميرة سمية للتكنولوجيا

Mid-term Exam
Duration (60) minutes
Grade /30
Date:17-4-2025

King Abdullah II school of Engineering – Department of Basic Sciences

Name: -- **ID** --------------------------

Q.1: (10 points)

Part(A) Determine whether each statement is TRUE or FALSE:

i- *If* $u = \langle u_1, u_2 \rangle$ *and* $v = \langle v_1, v_2 \rangle$ *then* $u.v = \langle u_1v_1, u_2v_2 \rangle$. (F)

ii- *The set of points* $\{(x,y,z)|x^2 + y^2 = 1\}$ *is a circle.* (F)

Part(B) Fill in the blanks with the CORRECT answers ONLY:

i) The distance from the point $P(-5,2,-3)$ *to the* $y-axis$ *is:* ($\sqrt{34}$)

ii) Let $f(x,y,z) = \ln\left(2z - \sqrt{x^2+y^2}\right)$, *and* $f(3,4,k) = 0$ *then* $k =$ (3)

iii) If $a.b = \sqrt{3}$, *and* $a \times b = \langle 1,2,2 \rangle$ *then the angle between a and b is:* ($\frac{\pi}{3}$ or 60°)

iv) A vector that has same direction as $6i + 2j - 3k$ *of length 14 is*($12i + 4j - 6k$)

Q.2: (3points)

Showing each step, calculate: $\underline{[(i+j) \times (i-j)]}.(k-i)$

$$\begin{vmatrix} i & j & k \\ 1 & 1 & 0 \\ 1 & -1 & 0 \end{vmatrix} = 0i - 0j - 2k$$

$$(-2k).(k-i)$$

$$\langle 0,0,-2 \rangle . \langle -1,0,1 \rangle = 0 + 0 - 2 = -2$$

Q.3: **(5 points)**

Part (A)

Show that $\lim_{(x,y)\to(0,0)} \frac{3xy}{x^2+y^2}$ *does not exit. (DO NOT USE POLAR COORDNITES)*

Along $\boxed{y=x}$

$$\lim_{x\to 0} \frac{3x^2}{x^2+x^2} = \lim_{x\to 0} \frac{3x^2}{2x^2}$$

$$= \frac{3}{2}$$

Along $\boxed{y=x^2}$

$$\lim_{x\to 0} \frac{3x^3}{x^2+x^4}$$

$$= \lim_{x\to 0} \frac{3x^3}{x^2(1+x^2)} = \lim_{x\to 0} \frac{3x}{1+x^2} = 0$$

Different Paths lead to different limiting values,
the given limit does not exist.

Part(B)

Use ***polar coordinates*** *to determine whether the following limit exits or not:*

$\lim_{(x,y)\to(0,0)} \frac{x^2y^2}{4x^2+4y^2}$ $\quad x = r\cos\theta \quad , \quad y = r\sin\theta \quad , \quad r^2 = x^2+y^2$

$$\lim_{(r,\theta)\to(0,\theta)} \frac{r^2\cos^2\theta . r^2\sin^2\theta}{4r^2} = \lim_{\substack{r\to 0\\ \theta\to\theta}} \frac{1}{4} r^2 \sin^2\theta \cos^2\theta$$

$$= \boxed{\text{Zero}} \qquad 0 \leq \sin^2\theta \leq 1$$
$$0 \leq \cos^2\theta \leq 1$$

Scanned with CamScanner

Q.4: (6 points)

Part(A) *Use implicit differentiation to find* $\frac{\partial z}{\partial y}$:

$e^{3z} = x^2y^3 \ln z$

$$3e^{3z} \cdot Z_y = 3x^2y^2 \ln z + x^2y^3 \frac{Z_y}{z}$$

$$3ze^{3z} Z_y = 3x^2y^2 z \ln z + x^2y^3 Z_y$$

$$Z_y = \frac{3x^2y^2 z \ln z}{3ze^{3z} - x^2y^3}$$

Part(B): *Given that* $z = \sqrt{x^2+y^2}$

i – *Identify the surface* $z = \sqrt{x^2+y^2}$

ii – *Find the equation of the tangent plane to* z *at* $(0, -1, 1)$.

i- The upper part of an open cone

ii- $Z_x = \frac{1}{2} \cdot \frac{2x}{\sqrt{x^2+y^2}} \rightarrow Z_x = \frac{1}{2} * \frac{0}{\sqrt{1}} = 0$

$Z_y = \frac{1}{2} \cdot \frac{2y}{\sqrt{x^2+y^2}} \rightarrow Z_y = \frac{1}{2} \cdot \frac{-2}{\sqrt{1}} = -1$

$$Z - 1 = 0(x-0) + -1(y+1)$$

$$Z = -y$$

Q.5 (6 points)

Part (A)

Given the points $A(4,1,-3), B(0,2,7),$ ***and*** $C(8,4,-1)$.

i)Find the point D which is the midpoint of the line segment from B to C.

ii)Find the parametric equations of the line L which passes through the point A and the point D.

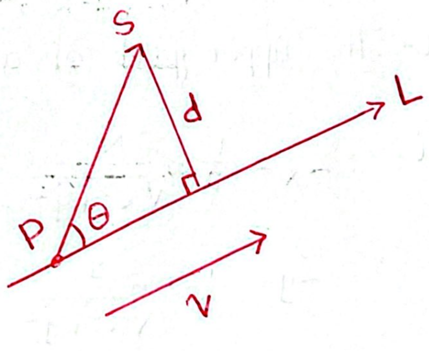
i) Midpoint $D = \left(\frac{0+8}{2}, \frac{2+4}{2}, \frac{7+-1}{2}\right) = (4,3,3)$

ii) $\overrightarrow{AD} = \langle 0,2,6 \rangle$

$L: \; x = 4$
$\quad y = 1+2t$
$\quad z = -3+6t$

Part(B)

Given that the line L passes through the point P and parallel to the vector $v = \langle a,b,c \rangle$***. Show that the distance (d) from the point S to the line L in the space is given by the formula:*** $d = \dfrac{|\overrightarrow{PS} \times \vec{v}|}{|\vec{v}|}$



- $\sin\theta = \dfrac{d}{|\overrightarrow{PS}|}$
- $|\overrightarrow{PS} \times \vec{v}| = |\overrightarrow{PS}||\vec{v}| \sin\theta$
- $\sin\theta = \dfrac{|\overrightarrow{PS} \times \vec{v}|}{|\overrightarrow{PS}||\vec{v}|}$
- $\dfrac{d}{|\overrightarrow{PS}|} = \dfrac{|\overrightarrow{PS} \times \vec{v}|}{|\overrightarrow{PS}||\vec{v}|}$

$d = \dfrac{|\overrightarrow{PS} \times \vec{v}|}{|\vec{v}|}$

END

GOOD LUCK